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Modelling Multiobjective Bilevel Programming for Environmental-Economic Power Generation and Dispatch using Genetic Algorithm

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Abstract. This article describes a multiobjective bilevel programming (MOBLP) model to solve environmental-economic power generation and dispatch (EEPGD) problem through genetic algorithm (GA) based fuzzy goal programming (FGP) in a thermal power plant operational system. In MOBLP formulation, first objectives of the problem are divided into two sets of objectives and assigned separately to two hierarchical decision levels (top-level and bottom -level) for optimization of them, where each level contains one or more controls variables associated with power generation decision system. Then, optimization problems of both the levels are described fuzzily to accommodate the impression arises for optimizing them in the decision situation. In FGP model formulation, the membership functions associated with defined fuzzy goals are designed, and then they are converted into membership goals by assigning highest membership value (unity) as achievement level and introducing under- and over-deviational variables to each of them. In goal achievement function, minimization of under-deviational variables of membership goals on the basis of weights of importance is considered to achieve optimal solution in the decision environment. In the process of solving the developed FGP model, a GA scheme is employed at two different stages, direct optimization of individual objectives at the first stage for fuzzy representation of them and, at the second stage, evaluation of goal achievement function to reach optimal power generation decision. The effective use of the method is demonstrated via IEEE 6-generator 30-bus System.

Keywords: Bilevel programming, Environmental-economic power generation, Fuzzy goal programming, Genetic algorithm, Membership function, Transmission-loss

1 Introduction

The major sources for electric power generation are thermal power plants, where more than 75% of them use coal to generate power with regard to meeting power demand in society. But, burning of fossil-fuel coal to generation power produces

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various harmful pollutants, namely oxides of carbon, nitrogen and sulphur and others.

It may be pointed out here that such by-products affect the entire living beings on earth. Therefore, the problem of EEPGD is essentially needed, where optimization of real-power generation cost and environmental pollution subject to various operational constraints have to be considered simultaneously to run thermal power plants.

Actually, thermal power plant operational problems in [1] are optimization problems with multiplicity of objectives in power generation decision environment. The mathematical programming (MP) model in power generation system was first studied by Dommel and Tinney in [2]. Thereafter, MP model for control of emission was discussed by Gent and Lament in [3]. Then, the field was further studied by Sullivan and Hackett in [4] and others to solve EEPGD problems .

However, the modelling aspect of minimizing both power generation cost and environmental-emission was initially introduced by Zahavi and Eisenberg in [5], and then the study on MP models for EEPGD problems was made in [6, 7] in the past.

A survey on the study of EEPGD problems made in the past was presented in [8] in 1977. Also, various MP models studied to solve EEPGD problems have been surveyed in [9, 10, and 11] in the past.

During 1990s, controlling of power plant emissions were considered seriously and different optimization methods in [12, 13, 14, 15, 16, 17] were presented with due consideration of 1990's Clean Air Amendment Act in [18] to make a pollution free environment. It is worthy to note here that the approaches to solve EEPGD problems with multiplicity of objectives made previously are classical ones on the line of transforming multiobjective models into single objective problems. As a matter of fact, decision troubles are frequently raised owing to the difficulty of taking individual optimality of objectives in decision making horizon.

Now, GP as efficient tool for multiobjecive decision analysis and based on satisficing philosophy in [19] has been employed to EEPGD problem in [20] to obtain goal oriented solution in crisp decision environment.

However, in most of the cases to model EEPGD problems, it may be noted that the parameters associated with objectives are not exact in nature due to imprecise nature of setting parameter values in actual practice. To overcome such a situation, fuzzy programming (FP) approaches in [21] have been introduced in [22, 23] to EEPGD problems in the past. Further, the use of stochastic programming (SP) to EEPGD problems has also been discussed in [24, 25] previously. But, extensive study on solving such problems is yet to circulate widely in literature.

Now it is worthy to mention that uses of classical approaches to MODM problems often leads to achieving suboptimal solution to owing to competing in nature of objectives in optimizing them as well as involvement of nonlinearity in objectives / constraints of a real-world problem. To avoid such a situation, GAs as a prominent tool in the area of nature-inspired computing can be used to solve MODM problems. The potential use of GAs to EEPGD problems have been discussed in [26, 27, 28] in the past.

Now, it is worth noting that the objectives of EEPGD problems are often conflicted regarding optimization of them in decision environment. As such, optimization of objectives in a hierarchical order can be taken into account and that is based on decision maker's (DM's) needs in the context of generation of thermal power. Therefore, optimization of them on the basis of hierarchical importance, and the use of bilevel programming (BLP) in [29] could be effective to reach optimal decision. Although, such a problem has been studied in [30] in the recent past, the area of study is at the initial stage. Again, MOBLP method to solve EEPGD problem within the framework of FGP by using GA is yet to circulate in literature.

In this article, an FGP method to solve MOBLP formulation of an EEPGD problem using GA is considered. In model formulation, *minsum* FGP in [31] as the simplest version of FGP is addressed to make power generation decision in fuzzy environment. In decision process, individual optimal decisions of objectives are determined first by using a GA scheme towards fuzzy goal description of objectives. Then, evaluation of goal achievement function as a second stage problem regarding minimization of weighted under-deviational variables of membership goals defined for fuzzy goals is considered. The effective use of the method is illustrated via IEEE 6-generator 30-bus System.

The paper is organized as follows. In Section 2, description of the problem is given. Section 3 contains MOBLP model formulation of EEPGD problem. In Section 4, the GA scheme for modelling and solving the problem is discussed. Section 5 provides the proposed FGP model of the problem. An illustrative case example is provided in Section 6. Finally, Section 7 highlights concluding remarks and scope for future research.

Now, objectives and constraints associated with EEPGD problem are discussed in the Sect. 2.

2 Problem Description

Let P_{gi} be the decision variables defined for generation of power (in p.u) from the *i*th generator of the system, $i = 1,2, ..., n$. Then, let P_D be total demand of power, T_L be total transmission- loss (in p.u) and P_L be the real power-loss in power generation system.

Then, objectives as well as constraints involved with the proposed EEPGD problem are presented in the following section.

2.1 Description of Objective Functions

The two types of objectives that are inherent to EEPGD model are presented as follows.

2.1.1 Economic Power Generation Objectives

a) Fuel-cost Function:

The total fuel-cost ($/h) incurred for of power generation can be expressed as:

$$F_C = \sum_{i=1}^{n} (a_i P_{gi}^2 + b_i P_{gi} + c_i) \ , \tag{1}$$

where a_i, b_i and c_i represent cost-coefficients concerning generation of power from *i*th generator.

b) Transmission-loss function:

The function associated with power transmission lines involves certain parameters which directly affect the ability to transfer power effectively. Here, the transmission-loss (T_L) (in p.u.) occurs during power dispatch can be modelled as a function of generator output and that can be expressed as:

$$T_L = \sum_{i=1}^{n}\sum_{j=1}^{n} P_{g_i} B_{ij} P_{g_j} + \sum_{i=1}^{n} B_{0i} P_{g_i} + B_{00} \ , \tag{2}$$

where B_{ij}, B_{0i} and B_{00} are *B*-coefficients in [23] associated with *i*-th generator in power transmission network.

2.1.2 Pollution Control Functions:

In a thermal power generation system, the most harmful pollutants that are discharged separately to earth's environment are oxides of nitrogen (NO_x), sulphur (SO_x) and carbon (CO_x). The pollution control functions are quadratic in nature and they are expressed in terms of generators' output P_{gi}, $i = 1,2,..., n$

The functional expression of total quantity of NO_x emissions (kg/h)) is of the form:

$$E_N = \sum_{i=1}^{n} d_{Ni} P_{gi}^2 + e_{Ni} P_{gi} + f_{Ni} \, , \tag{3}$$

where d_{Ni}, e_{Ni}, f_{Ni} represent NO_x emission-coefficients concerned with power generation from *i*th generator.

Similarly, the pollution control functions arise for SO_x- and CO_x-emissions appear as:

$$E_S = \sum_{i=1}^{n} d_{Si} P_{gi}^2 + e_{Si} P_{gi} + f_{Si} \, , \tag{4}$$

$$E_C = \sum_{i=1}^{n} d_{Ci} P_{gi}^2 + e_{Ci} P_{gi} + f_{Ci} \, , \text{ respectively,} \tag{5}$$

where the emission-coefficients associated with respective expressions can be defined in an analogous to the expression in (3).

2.2 Description of System Constraints

The constraints that are adhered to EEPGD problem are defined as follows.

2.2.1 Generator Capacity Constraints:

In thermal power generation system, the constraints on generators' outputs can be presented as:

$$P_{gi}^{min} \le P_{gi} \le P_{gi}^{max} \, , \qquad V_{g_i}^{min} \le V_{g_i} \le V_{g_i}^{max} \, , \ i = 1, 2, ..., n \tag{6}$$

where P_{gi} and V_{gi} represent active power and generator-bus voltage of *i*th generator, respectively.

2.2.2 Power Balance Constraint

The total power generated from the system must be equal to total demand (P_D) and total transmission-loss in thermal power generation system.

The power balance constraint takes the form:

$$\sum_{i=1}^{n} P_{gi} - (P_D + T_L) = 0 \qquad (7)$$

Now, formulation of MOBLP model of the problem is discussed in the Sect. 3.

3 MOBLP Formulation

In MOBLP formulation of the problem, the objectives concerning environmental-emission control are considered leader's optimization problems and that concerned with economic-power generation are considered follower's problems in hierarchical structure of EEPGD problem.

The MOBLP model is presented in the Sect. 3.1.

3.1 MOBLP Model

In the context of designing the proposed model, the vector of decision variables is divided into two distinct vector groups with regard to control them separately by DMs located at two hierarchical levels.

Let $\boldsymbol{X}$ be the vector of decision variables in power generation system. Then, let $\boldsymbol{X_L}$ and $\boldsymbol{X_F}$ be the subsets of $\mathbf{X}$ that are controlled by leader and follower, respectively, where L and F are used to denote leader and follower, respectively.

Then, MOBLP model can be stated as [29]:

Find $\boldsymbol{X}(\boldsymbol{X_L}, \boldsymbol{X_F})$ so as to:

$$\underset{X_L}{Minimize}\, E_N = \sum_{i=1}^{n} d_{N_i} P_{gi}^2 + e_{N_i} P_{gi} + f_{N_i},$$

$$\underset{X_L}{Minimize}\, E_S = \sum_{i=1}^{n} d_{S_i} P_{gi}^2 + e_{S_i} P_{gi} + f_{S_i},$$

$$\underset{X_L}{Minimize}\, E_C = \sum_{i=1}^{n} d_{C_i} P_{gi}^2 + e_{C_i} P_{gi} + f_{C_i},$$

(leader's problem)

and, for given $\boldsymbol{X_L}$, $\boldsymbol{X_F}$ solves

$$\underset{X_F}{Minimize}\ F_C = \sum_{i=1}^{n} (a_i P_{gi}^2 + b_i P_{gi} + c_i),$$

$$\underset{X_F}{Minimize}\, T_L = \sum_{i=1}^{n}\sum_{j=1}^{n} P_{g_i} B_{ij} P_{g_j} + \sum_{i=1}^{n} B_{0i} P_{g_i} + B_{00},$$

(follower's problem)

subject to the constraints in (6) and (7), (8)

where $\boldsymbol{X_L} \cap \boldsymbol{X_F} = \varphi$, $\boldsymbol{X_L} \cup \boldsymbol{X_F} = \boldsymbol{X}$ and $\boldsymbol{X} \in \mathrm{P}(\neq \varphi)$, where P denotes the feasible solution set, $\cap$ and $\cup$ stand for 'intersection' and 'union', respectively.

Now, the GA scheme adopted in the decision making environment is described in the Sect. 4.

4 GA Scheme

There is a variety of GA schemes in [32, 33] for generating new population by employing the 'selection', 'crossover' and 'mutation' operators.

In genetic search process, binary coded solution candidates are considered where initial population is generated randomly. The fitness of each chromosome (individual feasible solution) at each generation is justified with a view to optimizing objectives of the problem.

Now, formulation of FGP model of the problem in (8) is described in the Sect. 5.

5 FGP Model Formulation

In the structural framework of a BLP problem, it is conventionally considered that DM at each level is motivated to cooperative with other one concerning achievement of objectives in decision environment. In the sequel of making decision, since leader is with the power of making decision first, relaxation on his/her decision is essentially needed to make decision by follower to certain satisfactory level. Consequently, relaxation on individual objective values and components of $\boldsymbol{X_L}$ need be given to certain tolerance levels for benefit of follower. Therefore, use of the notion of fuzzy set to solve the problem in (8) would be effective one to reach overall satisfactory decision.
The fuzzy version of the problem is discussed in the Sect. 5.1.

5.1 Fuzzy Goal Description

In fuzzy environment, objective functions of the problem are to be expressed as fuzzy goals by means of incorporating an imprecise target value to each of them.

In the decision making context, since minimum value of an objective of a DM is highly acceptable, solutions achieved for minimization of objectives of individual DMs can be considered the best solutions, and they are determined as $(\boldsymbol{X}_L^{lb}, \boldsymbol{X}_F^{lb}; E_N^{lb}, E_S^{lb}, E_C^{lb})$ and $(\boldsymbol{X}_L^{fb}, \boldsymbol{X}_F^{fb}; F_C^{fb}, T_L^{fb})$, respectively, by employing GA scheme, where *lb* and *fb* indicate the best for leader and follower, respectively.

Then, the successive fuzzy goals take the form:

$$E_N \lesssim E_N^{lb}, \quad E_S \lesssim E_S^{lb} \text{ and } \quad E_C \lesssim E_C^{lb}$$

$$F_C \lesssim F_C^{fb} \quad \text{and} \quad T_L \lesssim T_L^{fb}, \tag{9}$$

where '$\lesssim$' indicates softness of $\le$ restriction and signifies 'essentially less than' in [34].

Again, since most dissatisfactory solutions of DMs correspond to maximum values of objectives, the worst solutions of leader and follower can be obtained by using the same GA scheme as $(\boldsymbol{X}_L^{lw}, \boldsymbol{X}_F^{lw}; E_N^{lw}, E_S^{lw}, E_C^{lw})$ and $(\boldsymbol{X}_L^{fw}, \boldsymbol{X}_F^{fw}; F_C^{fw}, T_L^{fw})$, respectively, where *lw* and, *fw* indicate worst cases for leader and follower, respectively.
As a matter consequence, $E_N^{lw}, E_S^{lw}, E_C^{lw}, F_C^{fw}$ and T_L^{fw} could be taken as upper-tolerance values towards achieving the respective fuzzy target levels E_N, E_S, E_C, F_C and T_L.

Again, fuzzy goal representation of control vector $\boldsymbol{X_L}$ can be reasonably taken as:

$$\boldsymbol{X_L} \lesssim \boldsymbol{X}_L^{lb} \tag{10}$$

Now, it may be mentioned that an increase in the value of a goal defined by goal vector in (10) would never be more than upper-bound of corresponding generator capacity defined in (6).
Let $\boldsymbol{X}_L^t, (\boldsymbol{X}_L^t < \boldsymbol{X}_L^{max})$, be the vector of upper-tolerance values to achieve the associated vector of fuzzy goal levels defined in (10).
Now, characterization of membership functions of fuzzy goals is described in the Sect. 5.2.

5.2 Characterization of Membership Function

The membership function of the fuzzy objective goal E_N can be algebraically presented as:

$$\mu_{E_N}\left[E_N\right]=\begin{cases} 1 \quad , & if\ E_N \le E_N^{lb} \\ \dfrac{E_N^{lw}-E_N}{E_N^{lw}-E_N^{lb}} \quad , & if\ E_N^{lb} < E_N \le E_N^{lw} \\ 0 \quad , & if\ E_N > E_N^{lw} \end{cases} \tag{11}$$

where $(E_N^{lw}-E_N^{lb})$ represents tolerance range for fuzzy goal achievement defined in (9).

Again, membership functions associated with other two objectives, E_s and E_c of leader as well as objectives of follower can be obtained.

The membership function associated with $\boldsymbol{X_L}$ can be obtained as:

$$\mu_{\boldsymbol{X_L}}\left[\boldsymbol{X_L}\right]=\begin{cases} 1, & if\ \ \boldsymbol{X_L} \le \boldsymbol{X_L^{lb}} \\ \dfrac{\boldsymbol{X_L^t}-\boldsymbol{X_L}}{\boldsymbol{X_L^t}-\boldsymbol{X_L^{lb}}}, & if\ \ \boldsymbol{X_L^{lb}} < \boldsymbol{X_L} \le \boldsymbol{X_L^t} \\ 0, & if\ \ \boldsymbol{X_L} > \boldsymbol{X_L^t} \end{cases} \tag{12}$$

where $(\boldsymbol{X_L^t}-\boldsymbol{X_L^{lb}})$ represents vector of tolerance ranges for achievement of vector of decision variables defined in (10). Now, *minsum* FGP model of the problem is presented in the Sect. 5.3.

5.3 *Minsum* FGP Model

To formulate FGP model of the problem, membership functions are converted into membership goals by assigning highest membership value (unity) as target level and introducing under- and over-deviational variables to each of them. In achievement function of *minsum* FGP model, minimization of the sum of weighted under-deviational variables associated with membership goals is taken into account.

The model appears as [31]:

Find $\boldsymbol{X}(\boldsymbol{X_L},\boldsymbol{X_F})$ so as to:

Minimize: $Z=\sum_{k=1}^{5} w_k^- d_k^- + \boldsymbol{w_6^- d_6^-}$

and satisfy

$$\frac{E_N^{lw}-E_N}{E_N^{lw}-E_N^{lb}}+d_1^- - d_1^+ = 1, \qquad \frac{E_S^{lw}-E_S}{E_S^{lw}-E_S^{lb}}+d_2^- - d_2^+ = 1,$$

$$\frac{E_C^{lw}-E_C}{E_C^{lw}-E_C^{lb}}+d_3^- - d_3^+ = 1, \qquad \frac{F_c^{fw}-F_C}{F_C^{fw}-F_C^{fb}}+d_4^- - d_4^+ = 1,$$

$$\frac{T_L^{fw}-T_L}{T_L^{fw}-T_L^{fb}}+d_5^- - d_5^+ = 1, \qquad \frac{\boldsymbol{X_L^t}-\boldsymbol{X_L}}{\boldsymbol{X_L^t}-\boldsymbol{P_{GL}^{lb}}}+\boldsymbol{d_6^-}-\boldsymbol{d_6^+} = \mathbf{I}$$

subject to the constraints in (6) and (7), (13)

where $d_k^-, d_k^+ \ge 0$, (k = 1,…,5) represent under- and over-deviational variables, respectively. $\boldsymbol{d_6^-}, \boldsymbol{d_6^+} \ge 0$ indicate vector of under- and over-deviational variables, respectively, and where I is a column vector. Z is goal achievement function, $w_k^- > 0$, k = 1, 2, 3, 4, 5 are relative numerical weights of importance of achieving target levels of goals, and $w_6^- > 0$ is the vector of numerical weights associated with d_6^-, and they are actually the inverse of respective tolerance ranges [31] concerning achievement of goal levels.

The effective use of the model in (13) is illustrated through a case example in the Sect. 6.

6 A Case Example

The IEEE 30-bus 6-generator test system in [15] is taken into account to demonstrate the proposed method.

The system is with 41 transmission lines and total power demand for 21 load buses is 2.834 p.u. The generator capacity limits and load data were discussed in [15] previously. The different types of coefficients associated with the model are given in Tables 1 - 4.

Table 1. Power generation cost-coefficients.

Generator →	g_1	g_2	g_3	g_4	g_5	g_6
Cost-Coefficients						
a	100	120	40	60	40	100
b	200	150	180	100	180	150
c	10	12	20	10	20	10

Table 2. NO_x emission-coefficients.

Generator →	g_1	g_2	g_3	g_4	g_5	g_6
NO_x Emission-Coefficients						
d_N	0.006323	0.006483	0.003174	0.006732	0.003174	0.006181
e_N	-0.38128	-0.79027	-1.36061	-2.39928	-1.36061	-0.39077
f_N	80.9019	28.8249	324.1775	610.2535	324.1775	50.3808

Table 3. SO_x emission-coefficients.

Generator →	g_1	g_2	g_3	g_4	g_5	g_6
SO_x Emission-Coefficients						
d_S	0.001206	0.002320	0.001284	0.000813	0.001284	0.003578
e_S	5.05928	3.84624	4.45647	4.97641	4.4564	4.14938
f_S	51.3778	182.2605	508.5207	165.3433	508.5207	121.2133

Table 4. CO_x emission-coefficients.

Generator →	g_1	g_2	g_3	g_4	g_5	g_6
CO_x Emission-Coefficients						
d_S	0.265110	0.140053	0.105929	0.106409	0.105929	0.403144
e_S	-61.01945	-29.95221	-9.552794	-12.73642	-9.552794	-121.9812
f_S	5080.148	3824.770	1342.851	1819.625	13.42.851	11381.070

The *B-coefficients* in [20] are presented as follows:

$$B = \begin{bmatrix} 0.1382 & -0.0299 & 0.0044 & -0.0022 & -0.0010 & -0.0008 \\ -0.0299 & 0.0487 & -0.0025 & 0.0004 & 0.0016 & 0.0041 \\ 0.0044 & -0.0025 & 0.0182 & -0.0070 & -0.0066 & -0.0066 \\ -0.0022 & 0.0004 & -0.0070 & 0.0137 & 0.0050 & 0.0033 \\ -0.0010 & 0.0016 & -0.0066 & 0.0050 & 0.0109 & 0.0005 \\ -0.0008 & 0.0041 & -0.0066 & 0.0033 & 0.0005 & 0.0244 \end{bmatrix}_{(6\times 6)}$$

$$B_0 = \begin{bmatrix} -0.0107 & 0.0060 & -0.0017 & 0.0009 & 0.0002 & 0.0030 \end{bmatrix}_{(1\times 6)}, \quad B_{00} = 9.86E-04$$

Now, to formulate MOBLP model, it is considered that $\boldsymbol{X_L}(P_{g3}, P_{g5})$ is under the control of leader, and $\boldsymbol{X_F}(P_{g1}, P_{g2}, P_{g4}, P_{g6})$ is that of follower.

Using the data presented in Tables 1- 4, the executable MOBLP model for EEPGD problem is stated as follows.

Find $X(P_{g1}, P_{g2}, P_{g3}, P_{g4}, P_{g5}\ P_{g6})$ so as to:

$$\underset{X_L}{Minimize}\ E_N(X) = (0.006323P_{g1}^2 - 0.38128P_{g1} + 80.9019 + 0.006483P_{g2}^2 - 0.79027P_{g2} + 28.8249$$
$$+ 0.003174P_{g3}^2 - 1.36061P_{g3} + 324.1775 + 0.006732P_{g4}^2 - 2.39928P_{g4} + 610.2535$$
$$+ 0.003174P_{g5}^2 - 1.36061P_{g5} + 324.1775 + 0.006181P_{g6}^2 - 0.39077P_{g6} + 50.3808) \quad (14)$$

$$\underset{X_L}{Minimize}\ E_S(X) = (0.001206P_{g1}^2 + 5.05928P_{g1} + 51.3778 + 0.002320P_{g2}^2 + 3.84624P_{g2} + 182.2605$$
$$+ 0.001284P_{g3}^2 + 4.45647P_{g3} + 508.5207 + 0.000813P_{g4}^2 + 4.97641P_{g4} + 165.3433$$
$$+ 0.001284P_{g5}^2 + 4.45647P_{g5} + 508.5207 + 0.003578P_{g6}^2 + 4.14938P_{g6} + 121.2133) \quad (15)$$

$$\underset{X_L}{Minimize}\ E_C(X) = (0.265110P_{g1}^2 - 61.01945P_{g1} + 5080.148 + 0.140053P_{g2}^2 - 29.95221P_{g2} + 3824.770$$
$$+ 0.105929P_{g3}^2 - 9.552795P_{g3} + 1342.851 + 0.106409P_{g4}^2 - 12.73642P_{g4} + 1819.625$$
$$+ 0.105929P_{g5}^2 - 9.552794P_{g5} + 1342.851 + 0.403144P_{g6}^2 - 121.9812P_{g6} + 11381.070)$$

(leader's objectives) (16)

and, for given X_L; X_F solve

$$\underset{X_F}{Minimize}\ F_C(X) = (100P_{g1}^2 + 200P_{g1} + 10 + 120P_{g2}^2 + 150P_{g2} + 10 + 40P_{g3}^2$$
$$+ 180P_{g3} + 20 + 60P_{g4}^2 + 100P_{g4} + 10 + 40P_{g5}^2 + 180P_{g5} \quad (17)$$
$$+ 20 + 100P_{g6}^2 + 150P_{g6} + 10)$$

$$\underset{X_F}{Minimize}\ T_L(X) = 0.1382P_{g1}^2 + 0.0487P_{g2}^2 + 0.0182P_{g3}^2 + 0.0137P_{g4}^2 + 0.0109P_{g5}^2 + 0.0244P_{g6}^2$$
$$- 0.0598P_{g1}P_{g2} + 0.0088P_{g1}P_{g3} - 0.0044P_{g1}P_{g4} - 0.0020P_{g1}P_{g5} - 0.0016P_{g1}P_{g6}$$
$$- 0.0050P_{g2}P_{g3} + 0.0008P_{g2}P_{g4} + 0.0032P_{g2}P_{g5} + 0.0082P_{g2}P_{g6} - 0.140P_{g3}P_{g4}$$
$$- 0.0132P_{g3}P_{g5} - 0.0132P_{g3}P_{g6} + 0.010P_{g4}P_{g5} + 0.0066P_{g4}P_{g6} + 0.0010P_{g5}P_{g6}$$
$$- 0.0107P_{g1} + 0.0060P_{g2} - 0.0017P_{g3} + 0.0009P_{g4} + 0.0002P_{g5} + 0.0030P_{g6} + 9.8573X10^{-4}$$

(follower's objectives) (18)

subject to , $0.05 \le P_{g1} \le 0.50,\quad 0.05 \le P_{g2} \le 0.60,$

$0.05 \le P_{g3} \le 1.00,\quad 0.05 \le P_{g4} \le 1.20,$

$0.05 \le P_{g5} \le 1.00,\quad 0.05 \le P_{g6} \le 0.60,$

(generator capacity constraints) (19)

and $P_{g1} + P_{g2} + P_{g3} + P_{g4} + P_{g5} + P_{g6} - (2.834 + L_T) = 0,$

(Power balance constraint) (20)

Now, in the GA scheme, 'Roulette-wheel selection' and 'single point crossover' with population size 50 are initially introduced. The parameter values adopted to execute the problem are crossover- probably = 0.8 and mutation- probability = 0.07.

The computer program developed in MATLAB and GAOT (Genetic Algorithm Optimization Toolbox) in MATLAB-Ver. R2010a is used to execute the problem. The execution is made in Intel Pentium IV with 2.66 GHz. Clock-pulse and 4 GB RAM.

Following the procedure, individual best solutions of leader and follower are found as:

$$(P_{g1}, P_{g2}, P_{g3}, P_{g4}, P_{g5}, P_{g6}; E_N^{lb}) = (0.05, 0.05, 0.5177, 1.20, 1.00, 0.05 \ ; 1413.708)$$

$$(P_{g1}, P_{g2}, P_{g3}, P_{g4}, P_{g5}, P_{g6}; E_S^{lb}) = (0.05, 0.60, 0.8379, 0.05, 0.7320, 0.60 \ ; 1549.535)$$

$$(P_{g1}, P_{g2}, P_{g3}, P_{g4}, P_{g5}, P_{g6}; E_C^{lb}) = (0.50, 0.60, 0.05, 1.0985, 0.05, 0.60 \ ; 24655.09)$$

$$(P_{g1}, P_{g2}, P_{g3}, P_{g4}, P_{g5}, P_{g6}; F_C^{fb}) = (0.1220, 0.2863, 0.5832, 0.9926, 0.5236, 0.3518 \ ; 595.9804)$$

$$(P_{g1}, P_{g2}, P_{g3}, P_{g4}, P_{g5}, P_{g6}; T_L^{fb}) = (0.0861, 0.0978, 0.9764, 0.5001, 0.8533, 0.3373 \ ; 0.0170).$$

Further, worst solutions of leader and follower are obtained as:

$$(P_{g1}, P_{g2}, P_{g3}, P_{g4}, P_{g5}, P_{g6}; E_N^{lw}) = (0.50, 0.60, 0.6036, 0.05, 0.5269, 0.60 \ ; 1416.167)$$

$$(P_{g1}, P_{g2}, P_{g3}, P_{g4}, P_{g5}, P_{g6}; E_S^{lw}) = (0.50, 0.05, 0.1002, 1.2, 1.00, 0.05 \ ; 1551.043)$$

$$(P_{g1}, P_{g2}, P_{g3}, P_{g4}, P_{g5}, P_{g6}; E_C^{lw}) = (0.05, 0.05, 1.00, 0.7040, 1.0\,0, 0.05 ; 2475286)$$

$$(P_{g1}, P_{g2}, P_{g3}, P_{g4}, P_{g5}, P_{g6}; F_C^{fw}) = (0.500, 0.600, 0.13\,97, 0.05, 1.\,00, 0.600 ; 705.2694)$$

$$(P_{g1}, P_{g2}, P_{g3}, P_{g4}, P_{g5}, P_{g6}; T_L^{fw}) = (0.50, 0.05, 0.05, 1.20, 1.00, 0.1036 \ ; 0.0696)$$

Then, the fuzzy objective goals are obtained as:

$E_N \lesssim 1413.708$, $E_S \lesssim 1549.535$, $E_C \lesssim 24655.09$, $F_C \lesssim 595.9804$, $T_L \lesssim 0.0170$.

The fuzzy goals for power generation decisions under the control of leader appear as:

$$P_{g3} \lesssim 0.15 \text{ and } P_{g5} \lesssim 0.15 .$$

The upper-tolerance limits of E_N, E_S, E_C, F_C and T_L are obtained as $(E_N^{lw}, E_S^{lw}, E_C^{lw}, F_C^{fw}, T_L^{fw}) = (1416.167, 1551.043, 24752.86, 705.2694, 0.0696,)$. Again, the upper-tolerance limits of the decision variables associated with $\mathbf{X_L}$ are considered $(P_{g3}^t, P_{g5}^t) = (0.6, 0.6)$.

Then, the membership functions are constructed as follows:

$$\mu_{E_N} = \frac{1416.167 - E_N}{1416.167 - 1413.708}, \quad \mu_{E_S} = \frac{1551.043 - E_S}{1551.043 - 1549.535}, \quad \mu_{E_C} = \frac{2475286 - E_C}{2475286 - 2465509}, \quad \mu_{F_C} = \frac{705.2694 - Z_1}{705.2694 - 595.9804},$$

$$\mu_{T_L} = \frac{0.0696 - T_L}{0.0696 - 0.0170},$$

$\mu_{P_{g3}} = \dfrac{0.60 - P_{g3}}{0.60 - 0.40}$, $\quad \mu_{P_{g5}} = \dfrac{0.60 - P_{g5}}{0.70 - 0.40}$

Then, the executable *minsum* FGP model is constructed as follows.

Find $\boldsymbol{X}(P_{g1}, P_{g2}, P_{g3}, P_{g4}, P_{g5}, P_{g6})$ so as to:

Minimize $Z = 0.4067 d_1^- + 0.6631 d_2^- + 0.0102 d_3^- + 0.0092 d_4^- + 19.0114 d_5^- + 2.5 d_6^- + 2.5 d_7^-$

and satisfy

$$\frac{1416.167 - \sum_{i=1}^{n} d_{Ni} P_{gi}^2 + e_{Ni} P_{gi} + f_{Ni}}{1416.167 - 1413.708} + d_1^- - d_1^+ = 1$$

$$\frac{1551.043 - \sum_{i=1}^{n} d_{Si} P_{gi}^2 + e_{Si} P_{gi} + f_{Si}}{1551.043 - 1549.535} + d_2^- - d_2^+ = 1$$

$$\frac{2475286 - \sum_{i=1}^{n} d_{Ci} P_{gi}^2 + e_{Ci} P_{gi} + f_{Ci}}{2475286 - 2465509} + d_3^- - d_3^+ = 1$$

$$\frac{705.2694 - \sum_{i=1}^{n} (a_i P_{gi}^2 + b_i P_{gi} + c_i)}{705.2694 - 595.9804} + d_4^- - d_4^+ = 1$$

$$\frac{0.0696 - \sum_{i=1}^{n}\sum_{j=1}^{n} P_{g_i} B_{ij} P_{g_j} + \sum_{i=1}^{n} B_{0i} P_{g_i} + B_{00}}{0.0696 - 0.0170} + d_5^- - d_5^+ = 1$$

$$\frac{0.60 - P_{g3}}{0.60 - 0.40} + d_6^- - d_6^+ = 1, \quad \frac{0.60 - P_{g5}}{0.60 - 0.40} + d_7^- - d_7^+ = 1$$

subject to the constraints in (19) and (20). (21)

The function Z in (21) acts as evaluation function in solution search process.
The function to evaluate the fitness of a chromosome takes the form:

$$Eval(E_v) = (Z)_v = \left(\sum_{k=1}^{5} w_k^- d_k^- + \sum_{k=6}^{7} w_k^- d_k^-\right)_v, \; v = 1, 2, \ldots, PS,$$

where *PS* stands for population-size. (22)

where $(Z)_v$ represents the achievement function (Z) to measure fitness value of vth chromosome.

The best objective value (Z^*) at any solution stage is obtained as:

$$Z^* = \min\{eval(E_v) \mid v = 1, 2, \ldots, PS\} \quad (23)$$

The resultant objective values are found as:

$(E_N, E_S, E_C, F_C, T_L) = (1414.69, 1550.38, 24669.95, 62973, 0.0522)$

with the respective membership values:

$(\mu_{E_N}, \mu_{E_S}, \mu_{E_C}, \mu_{F_C}, \mu_{T_L}) = (0.5978, 0.4357, 0.8479, 0.6912, 0.0255)$.

The power generation decision is obtained as:

$(P_{g1}, P_{g2}, P_{g3}, P_{g4}, P_{g5}, P_{g6}) = (0.1821, 0.4197, 0.40, 0.9885, 0.40, 0.47737)$.

The bar-diagram to represent power generation decision is depicted in Figure 1.

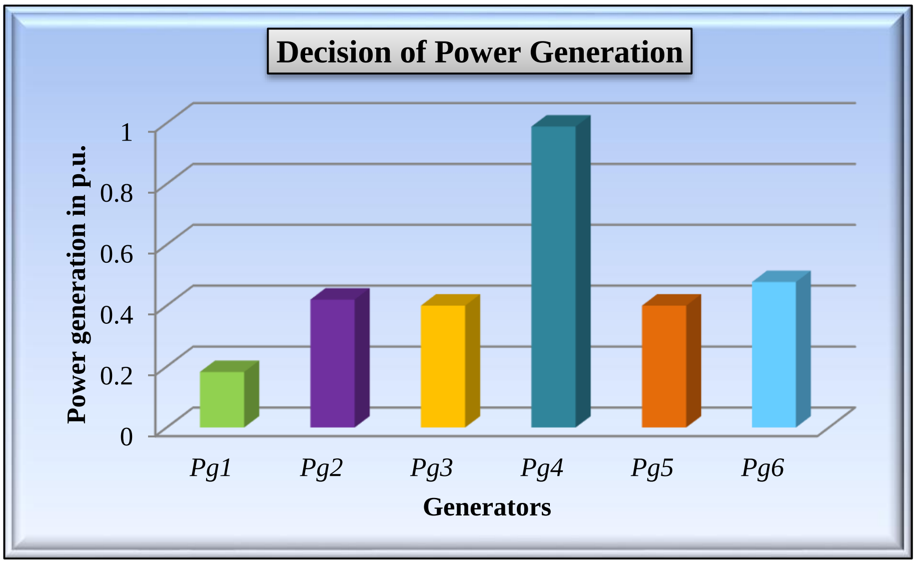

Fig. 1. Graphical representation of power generation decision.

The result indicates that the solution is quite satisfactory, and sequential executions of decision powers of DMs are preserved there in the hierarchical order of optimizing objectives of the EEPGD problem.

7 Performance Comparison

To highlight more the effectiveness of the proposed method, a comparison of resultant solution is made with the solution achieved by employing the conventional *minsum* FGP method in [35].

Here, values of the objectives are found as:

$$(E_N, E_S, E_C, F_C, T_L) = (1414.847, 1550.01, 24719.38, 63160, 0.0175).$$

The resultant power generation decision is:

$$(P_{g1}, P_{g2}, P_{g3}, P_{g4}, P_{g5}, P_{g6}) = (0.05, 0.1409, 0.9898, 0.4379, 0.8938, 0.3389).$$

The above result indicates that reduction of 49.43 kg/hr of NO_x emission and reduction of 1.87 $/hr fuel cost are made here by using the proposed method without sacrificing total units of power demand.

8 Conclusions and Future Research Direction

The main advantage of using BLP to EEPGD problem is that optimization of objectives individually in a hierarchical order can be obtained in inexact environment. Again, order of hierarchy of objectives as well as fuzzy descriptions of objectives / constraints can easily be rearranged under the flexible nature of the proposed FGP model in decision making horizon. Furthermore, computational burden arises with linearization of objectives by using conventional technique does not involve here owing to the use of bio-inspired tool to make power generation decision. Here, it may be claimed that the GA based FGP method presented here may open up future research for thermal power generation decision and to make pollution free living environment on earth. However, the proposed method can be extended to formulate multilevel programming (MLP) [36] model with multiplicity of objectives in power plant operation and management system, which is an emerging problem in future research.

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